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Public education, communities and vouchers

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Abstract

This paper examines the effects of school vouchers on the equilibrium of a simple two-community model with local provision of education. Individuals differ only in income and one community imposes a fixed cost on its residents. We show that the introduction of a simple voucher system can result in a Pareto improvement as an equilibrium phenomenon. This improvement brings higher education quality to both communities.

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1. Introduction

While nearly 90% of all school-aged children in the US attend government-owned schools, more and more parents are dissatisfied with the perceived quality of their children's public schools. Overall, urban schools are apparently failing to educate a large proportion of their inner-city students.

As Poterba (1994) has pointed out, one of the standard justifications of government intervention in private markets is *redistribution*: education, it is argued, is a fundamental right which should not be allocated according to ability to pay. But if parental resources are unequal, a well functioning private market for education will result in differences in the quality of education

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that children receive. These differences in education imply differences in earning opportunities, which may be seen as unfair since they are beyond the child's control.

For historical reasons, most education services in the US are provided locally and funded by local property taxes. Since communities differ in their tax bases and willingness to pay, levels of spending on education differ greatly across communities and redistribution is limited. Rich parents can choose between public and private schools for their children, and the community in which they reside. In contrast, poor parents must send their children to the nearest public school, whether or not this is educationally effective or even safe.

Consider the example of Rhode Island. There are 36 school districts in the state of Rhode Island. Due to space considerations, we only show a few districts: The city of Providence, two wealthy communities (Barrington and East Greenwich), and a poorer community (East Providence). Table 1 presents the relevant information.

Table 1
Some education statistics of Rhode Island

District	Rhode Island	Providence	Barrington	East Greenwich	East Providence
Resources					
Median family income ($)	39,172	28,342	59,483	61,843	37,634
Per capita income ($)	14,981	11,838	24,965	26,163	14,387
Property value per pupil ($)	309,319	144,233	565,523	603,171	316,927
Property tax rate (%)	–	30.42	20	22.15	35.75
Students					
Total number	148,977	29,197	3,363	2,691	8,091
Public enrollment (%)	85.35	82.44	85.61	83.39	84.32
Minority (%)	21	75	2	4	14
White	79	25	98	96	86
Private enrollment (%)	14.37	17.36	13.97	16.57	15.49
Other enrollment (%)	0.28	0.20	0.42	0.19	0.19
Revenues					
Local (%)	57.12	36.24	88.53	91.39	58.20
State (%)	39.07	57.55	9.4	7.42	38.87
Federal (%)	3.81	6.21	2.07	1.19	2.93
Expenditures					
Per pupil ($)	7,233	7,259	7,106	6,911	6,556
General instruction ($)	3,781	3,107	4,027	4,008	3,661
Results					
Graduation rate	82.7	75.2	95.2	92.3	77.7
Drop-out rate (%)	17.3	24.8	4.8	7.7	22.3
Seniors taking SAT (%)	64	64	97	96	59
Math score	489	438	533	536	464
Verbal score	495	442	522	539	472
Total score	984	880	1,055	1,075	936

Sources: Income and property values from *Rhode Island Public Schools: 1996 District Profiles*, published by *The Department of Education of the State of Rhode Island and Providence Plantations*, henceforth RIPS. Tax rate from *Providence Journal Almanac, 1997.*

First, there are important differences in median income, income per capita, tax base and tax rates across communities. Note that richer communities (in terms of median income or income per capita) have larger tax bases and lower tax rates.

Second, the students are different in the different communities. Note that (i) richer communities have smaller student populations; (ii) there is no big difference in the proportion of students enrolled in public and private schools; (iii) there are sizable differences in the proportions of white and minority students across communities: in Providence, 75% of the students in public schools belong to a minority group, in East Greenwich the share is 4% and in Barrington it is only 2%.

Third, local resources are specially important for richer communities. Barrington and East Greenwich finance education mainly out of local tax revenues, while the other communities use both local and state funds. Federal funds play a minor role in all cases. Secondly, there is no big difference in total expenditure per pupil. However, there are some differences in expenditure devoted to general instruction (as opposed to administrative support, non-instructional services to students, transportation, management of facilities, etc.): richer communities spend more.

Lastly, students leaving the system perform differently. In richer communities graduation rates are much higher and dropout rates are much lower. A higher proportion of seniors take the SAT and their average scores are higher.

It should be clear from the above table that "other things equal" any reasonable parent living in Providence would rather live in Barrington or East Greenwich, where expenditure per pupil is similar to that in Providence but where the quality of education, as shown in Table 1, is much higher. If some residents of Providence use its public education system, there must be some "moving costs" (e.g., zoning regulations, transportation costs to work, or amenities), which render the education offered by the other cities too costly. It seems that the present situation is not efficient: a parent residing in Providence could be given the local per pupil expenditure and be allowed to use it to finance his child's education in Barrington, without relocating there. This would constitute a Pareto improvement.

This fact has not been overlooked by many frustrated parents who are looking at possible alternatives to the US educational system. Some proposed reforms imply one form or another of school vouchers, all of which involve disbursing public money to help parents pay for the cost of sending their children to the school of their choice, typically a school other than the district public school. It has been argued that school vouchers will improve education by inducing competition among schools to attract students. With guaranteed enrollment rates and a low fraction of parents with the necessary means to opt out of the system, public schools have little incentive to provide good education. Friedman (1962) argues that vouchers may result in a more efficient use of public resources in education.

However, there are also equity implications to vouchers. School vouchers could exacerbate economic, racial or ability segregation. If the voucher is less than needed to cover tuition in a private school, only relatively rich parents could use them, leaving the poorest children in the public schools, with perhaps even less resources. In addition, private schools are not obliged to accept all applicants—they may accept only the smart ones. This would have an adverse effect on the students staying in the public school system if one believes that peer effects are an important determinant of the quality of education.[1]

Although several voucher experiments are currently underway, e.g., in Minnesota, Wisconsin, New York, and Florida, empirical evidence is still scarce. Hoxby (1994) suggests that private school competition would greatly increase public school effectiveness. Figlio and Stone (1997) challenge the finding that private schools always do better.

A theoretical approach to the subject has proven difficult. Ideally, one would like an integrated framework of community choice, housing and schooling, with agents differing by a variety of characteristics including, in addition to income, ability and race. Moreover, the framework should be appropriate to study the efficiency and equity effects of vouchers discussed above. Needless to say, most models abstract from some relevant issues and focus on some others. In many cases, the theoretical literature relies on computational methods to produce examples or to calibrate models to the available data. One strand of literature assumes that the quality of public education is homogeneous across communities and analyzes the effect of school vouchers that allow children to attend private schools (for example, Epple & Romano, 1998; Hoyt & Lee, 1998; Ireland, 1990; Manski, 1992). A second strand of literature works in a multi-community framework and again analyzes the effects of school vouchers that can be used to attend private schools (for example, Nechyba, 1996, 1999; Rangazas, 1995). In this paper, we analyze the effects of school vouchers that can be used in other public schools in a framework where the quality of education may differ across public schools located in different communities.[2]

The aim of the paper is to show, within a two-community model, that a simple voucher system can result in a Pareto improvement as an equilibrium phenomenon. By this we mean that the voucher system will result in an equilibrium outcome that Pareto dominates the equilibrium outcome in the absence of vouchers. Since we want to compare equilibria with and without vouchers, we choose to work with a very simple model similar in spirit to that of Westhoff (1977) and Fernandez and Rogerson (1996). There is a continuum of agents, characterized by income level, who must be allocated into one of two communities. Each community is characterized by a proportional income tax rate which is used to finance locally provided public education. Education is produced by means of a non-increasing returns to scale production function and the tax rate is determined by majority vote within each community. An additional feature of our model is that one community, say Barrington, imposes a fixed cost on its residents. This cost may represent the fact that the other city, Providence, offers more amenities, or that people work in Providence and hence there is a transportation cost involved in living in Barrington.[3] In equilibrium, agents are unwilling to move from one city to another. As it is typical in these models, equilibria will be stratified and education quality will be higher in the rich community. Further, as a result of the fixed cost, under reasonable assumptions on the agents' preferences and education technology, the rich community will charge a lower tax rate. Some examples in Fernandez and Rogerson (1997) show this high quality—low tax pattern, but this paper shows some sufficient conditions for it to hold.

After calculating an example of equilibrium, we consider the introduction of a voucher system whereby the residents of the poorer community can get the local per pupil expenditure on education and use it to finance their children's education in the richer community, without having to relocate. Since the resulting flow of students imposes a negative externality on the absorbing community, it is required that the incoming students pay a sufficiently high price for education to maintain its original quality. In other words, the level of education of the rich community should remain unaffected after allowing the other community's children access to the local school. We calculate several equilibria under this voucher system and show that some of them result in a Pareto improvement with respect to the equilibrium without vouchers. In these equilibria, education quality increases in both communities, and some people move from the rich community to the poor one. This last fact is consistent with the findings of Fernandez and Rogerson (1996), who suggest that "policies whose net effect is to increase the number of residents in the poorest community will tend to be Pareto improving."

The rest of the paper is organized as follows. Section 2 introduces the benchmark model without vouchers and presents an example. Section 3introduces vouchers and shows how they can result in a Pareto improvement. Section 4 concludes. All the proofs are relegated to Appendix A.

2. The economy

2.1. The benchmark model

The economy has two goods: a private good (money) and a locally provided public good (education). There is a continuum of agents, represented by the closed unit interval [0,1] denoted by I, which is endowed with the Lebesgue measurable sets and the Lebesgue measure λ on them. Individuals have identical utility functions:

$$u(x, q), \tag{1}$$

where x is the quantity of the private good and q is the quality of the education consumed. Individuals differ in their initial endowment of the private good, which is represented by the Lebesgue integrable function

$$y : I \to \mathbb{R}_+. \tag{2}$$

The quantity $y(i)$ represents the amount of money owned by agent i. The distribution of the initial endowments of the private good is represented by the cumulative distribution function, $F : \mathbb{R}_+ \to [0, 1]$ defined by

$$F(y_0) = \lambda[y^{-1}([0, y_0])], \tag{3}$$

where y^{-1} is the inverse of y in Eq. (2). $F(y)$ represents the fraction of agents with initial endowment equal to or lower than y.

There are two cities: Providence (P) and Barrington (B). Living in each city entails a fixed cost c_J, where $J = P, B$. We assume that $c_P = 0$ and $c_B = c > 0$. We assume a fixed exogenous cost for simplicity. We could alternatively choose an endogenous cost which depended on the number of people living in the community: the smaller the community, the higher the fixed cost of living there. The qualitative results of our paper would be unchanged. Depending on the context, the index J will denote the name of a city or the set of that city's residents.

Education is produced in each city using the private good as an input, and according to the production function, g,

$$g : \mathbb{R}_+ \to \mathbb{R}_+. \tag{4}$$

If E_J is the amount of input devoted to education in city J, then $g(E_J)$ is the total amount of education produced. The quality of education received by an agent living in that city is

$$\frac{g(E_J)}{\lambda(J)}, \tag{5}$$

where $\lambda(J)$ is the population measure of city J. Note that if g is the identity function, then the quality of education equals average expenditure on education in the community, which is the typical specification in the literature. Thus, our specification encompasses the standard assumption that the quality of education depends only on expenditure per capita. As long as g exhibits decreasing returns, there will be some crowding effect. One may think of these decreasing returns as arising from the existence of some fixed factor in the production of education. This assumption is not a necessary condition for our main conclusions to hold. It just facilitates the construction of equilibria with two distinct communities of positive size.

The primitives of the model are then the preferences, the income distribution, the education technology and the cost of living in Barrington. Therefore, a typical economy will be denoted by $\mathcal{E} = \langle u, y, g, c\rangle$. After describing the economy, we can turn to its organization. Cities are restricted to finance the provision of education *via* proportional income taxation. Therefore, for city J with population measure $\lambda(J)$ and tax rate t, the quality of education is

$$q(t, J) = \frac{g\left(t\int_J y(i)\mathrm{d}\lambda\right)}{\lambda(J)}. \tag{6}$$

Eq. (6) is the *budget constraint* of city J.

Definition 2.1. A community (J, t_J, q_J) is a measurable subset $J \subseteq I$ of positive measure, an income tax rate $t_J \in [0, 1]$, and a quality of education, q_J, such that

$$q_J = q(t_J, J) \equiv \frac{g\left(t_J\int_J y(i)\mathrm{d}\lambda\right)}{\lambda(J)}, \tag{7}$$

i.e., q_J satisfies the budget constraint. We say that community (J, t_J, q_J) is viable if for all $j \in J$, $(1 - t_J)y(j) - c_J \geq 0$.

In this model, we require cities not only to finance public education *via* a proportional income tax, but also to decide on the level of taxation by majority vote.

Definition 2.2. Let (J, t_J, q_J) be a viable community. Let $t \in [0, 1]$ be a tax rate and let $q(t, J)$ be the education quality associated with it according to the budget constraint. Let the inhabitants of J who prefer tax rate t to t_J be denoted by $J(t)$:

$$J(t) = \{i \in J : u[(1 - t)y(i) - c_J, q(t, J)] > u[(1 - t_J)y(i) - c_J, q_J]\}. \tag{8}$$

We say that (J, t_J, q_J) is in a majority voting equilibrium if

$$\lambda(J(t)) \leq \tfrac{1}{2}\lambda(J), \quad \forall t \in [0, 1]. \tag{9}$$

We are now ready to define the equilibrium concept that we will use in the first part of the paper.

Definition 2.3. An equilibrium of the economy $\mathcal{E} = \langle u, y, g, c\rangle$ is a partition $\langle(P, t_P, q_P), (B, t_B, q_B)\rangle$ of I into two viable communities that satisfies:

1. Majority voting. Both (P, t_P, q_P) and (B, t_B, q_B) are in a majority voting equilibrium.
2. Free mobility. No agent prefers the other community to his own:

$$\begin{aligned} &u[(1-t_P)y(i), q_P] \geq u[(1-t_B)y(i)-c, q_B], \quad \forall i \in P,\\ &u[(1-t_B)y(i)-c, q_B] \geq u[(1-t_P)y(i), q_P], \quad \forall i \in B. \end{aligned} \tag{10}$$

We define an equilibrium as a partition into two communities—thus not allowing for an equilibrium with only one community—for analytical convenience. It can be checked however, that when the production function of education satisfies $\lim_{x\to 0} g'(x) = \infty$, the quality of education becomes arbitrarily large as the community becomes small, as long as tax collection is non-negative. This implies that any situation with only one community would be unstable.

2.2. Some useful propositions

We now present some general facts that will help us calculate equilibria later. Propositions 1 and 2 guarantee that the rich and the poor separate, and that there exists an agent who is indifferent between living in Providence or Barrington.

Proposition 1 (Stratification). *Let $\mathcal{E}$ be an economy and let $\langle(P, t_P, q_P), (B, t_B, q_B)\rangle$ be an equilibrium of $\mathcal{E}$. Define*

$$v(y) = u[(1-t_P)y, q_P] - u[(1-t_B)y - c, q_B], \tag{11}$$

as the utility differential between living in Providence and living in Barrington for an agent with initial endowment of the private good y.

There is income stratification—the poor live in Providence and the rich live in Barrington—if

$$\frac{\mathrm{d}v(y)}{\mathrm{d}y} < 0, \qquad \forall y \in [y_l, y_h]. \tag{12}$$

There is income stratification—the poor live in Barrington and the rich live in Providence—if

$$\frac{\mathrm{d}v(y)}{\mathrm{d}y} > 0, \qquad \forall y \in [y_l, y_h]. \tag{13}$$

Proposition 2 (Boundary indifference). *Let $\mathcal{E} = \langle u, y, g, c\rangle$ be an economy where u is a continuous function and assume that the range of y is a convex set. Let $\langle(P, t_P, q_P), (B, t_B, q_B)\rangle$ be a stratified equilibrium of $\mathcal{E}$. Then there is an individual i who is indifferent between the two equilibrium communities.*

Propositions 3 and 4 below ensure that the median voter in each community decides the tax rate and that the median voter is the agent with median income.

Proposition 3 (Single peaked preferences). *Let $u(x, q)$ be a strictly concave, continuous function with $u_{12} \geq 0$. Let $g(z)$ be a concave and continuous production function, with $g' > 0$ and $g'' \leq 0$. Then preferences are single peaked on t.*

Proposition 4 (Monotonicity). *Let (J, t_J, q_J) be a community where t_J is the most preferred tax rate of the individual with median income. If u_2/yu_1 is a monotonic function of y, then (J, t_J, q_J) is in a majority voting equilibrium.*

It is worth pointing out that Proposition 1, which guarantees stratification in our model, is different from the "single-crossing" property used in Westhoff (1977) and in Fernandez and Rogerson (1996), which guaranteed stratification there. The assumption there required the slope of the indifference curve of an agent in the (q, t) space to be increasing in initial income. We do not require this, as we will see in the example that follows, where this slope is constant. Therefore, our model generates stratification in some cases that are not covered by the above models. Note also that Proposition 4, which guarantees that the median voter is the agent with median income, only requires this slope to be monotonic. In their case this should be increasing.

2.3. Example

We now present an example that illustrates the model introduced in the previous section and the solutions that one can obtain with this simple set-up.

Let $\mathcal{E}_1$ be an economy where the common utility function is $u(x, q) = \ln(x) + \ln(q)$, $g(z) = z^\alpha, 0 < \alpha < 1$. Note that g exhibits decreasing returns. Further assume that y is a continuous function with range $[y_l, y_h] \subseteq \mathbb{R}_{++}$. We introduce some propositions that will help us characterize and calculate the equilibrium values for this particular example.

Proposition 5.

1. *Preferences on t are single peaked.*
2. *In equilibrium, the median voter of each community is the agent with median income.*
3. *In every equilibrium of $\mathcal{E}_1$, there is income stratification, the rich live in Barrington and the poor live in Providence.*
4. *In every equilibrium there is an individual who is indifferent between the two communities.*

The following proposition shows that in this class of economies, the rich community imposes a lower tax rate while providing a better quality of education. Some examples in Fernandez and Rogerson (1997) show this pattern but some others do not. Within the class of economies under consideration, this pattern is a general result.

Proposition 6. *In every equilibrium,*

1. *$t_B < t_P$. The tax rate in Barrington is lower than the tax rate in Providence.*
2. *$q_B > q_P$. The quality of public education in Barrington is higher than the quality of public education in Providence.*

We can now use Proposition 5 to derive a system of equations that determines an equilibrium of the economy. The first equation identifies the agent, i_b, with endowment y_b, who is indifferent between the two communities:

$$u[(1 - t_P)y_b, q_P] = u[(1 - t_B)y_b - c, q_B].$$

Using stratification, the next two equations determine the median income in Providence and in Barrington, denoted by y_P^m, and y_B^m, respectively.

$$\frac{1}{2} = \frac{\int_{y_l}^{y_P^m} \mathrm{d}F}{\int_{y_l}^{y_b} \mathrm{d}F} \quad \text{and} \quad \frac{1}{2} = \frac{\int_{y_B^m}^{y_h} \mathrm{d}F}{\int_{y_b}^{y_h} \mathrm{d}F}.$$

Since preferences are single peaked, the tax rate is the one preferred by the median voter, who (by Proposition 5) is the agent with median income in that community.

$$t_P = \arg\max_t u\left[(1-t)y_P^m, \frac{g\left(t\int_{y_l}^{y_b} y\,\mathrm{d}F\right)}{\int_{y_l}^{y_b} \mathrm{d}F}\right],$$

$$t_B = \arg\max_t u\left[(1-t)y_B^m - c, \frac{g\left(t\int_{y_b}^{y_h} y\,\mathrm{d}F\right)}{\int_{y_b}^{y_h} \mathrm{d}F}\right].$$

In each community, the budget constraints must be satisfied, which determines the quality of education:

$$q_P = \frac{g\left(t_P\int_{y_l}^{y_b} y\,\mathrm{d}F\right)}{\int_{y_l}^{y_b} \mathrm{d}F}, \qquad q_B = \frac{g\left(t_B\int_{y_b}^{y_h} y\,\mathrm{d}F\right)}{\int_{y_b}^{y_h} \mathrm{d}F}.$$

In order to find an equilibrium, we now have to solve the former system of seven equations with seven unknowns $(y_b, y_P^m, y_B^m, t_P, t_B, q_P, q_B)$.[4] We calculate the equilibrium when $\alpha = 1/2$, $i \in [0, 1]$, and $y = 9i + 1$ (or equivalently $y \sim U[1, 10]$) for different values of c, the fixed cost of living in Barrington. The results are summarized in Table 2.

Following Proposition 5, there is income stratification with the rich living in Barrington and the poor living in Providence. We observe that as c increases, Barrington becomes a more exclusive community and the indifferent agent, i_b, becomes richer (y_b increases). The tax rate in Providence, t_P, is higher than the tax rate in Barrington, t_B (Proposition 6). Note that as c increases, the tax rate in Barrington decreases while the tax rate in Providence does not change.

Table 2
Equilibrium values without vouchers

c	i_b	y_b	t_B	q_B	e_B	t_P	q_P	e_P	y_0
0	0.2403	3.1623	0.3333	1.6992	2.1937	0.3333	1.6992	0.6937	–
1	0.4404	4.9632	0.2888	1.9648	2.1605	0.3333	1.5023	0.9938	–
2	0.5873	6.2857	0.2515	2.2274	2.0476	0.3333	1.4379	1.2143	–
3	0.7021	7.3191	0.2178	2.5166	1.8865	0.3333	1.4052	1.3865	–
4	0.7939	8.1448	0.1864	2.8640	1.6908	0.3333	1.3856	1.5241	7.4869
5	0.8668	8.8008	0.1560	3.3179	1.4668	0.3333	1.3727	1.6335	7.5894
6	0.9228	9.3050	0.1261	3.9707	1.2175	0.3333	1.3642	1.7175	7.8556
7	0.9629	9.6654	0.0960	5.0397	0.9442	0.3333	1.3587	1.7775	8.3165

For this table: $u(x, q) = \ln(x) + \ln(q)$, $g(z) = z^\alpha$, $\alpha = 1/2$, $y \sim U[1, 10]$.

Consistent with the same proposition, the quality of education in Barrington, q_B, is higher than the quality of education in Providence, q_P, except when $c = 0$, in which case both cities provide the same quality. We also observe that as c increases, the quality of education improves considerably in Barrington, while it deteriorates in Providence. Expenditure per student, e_B, starts higher in Barrington but decreases as c increases and sinks below the corresponding expenditure per student in Providence, e_P. These results contrast with part of the previous literature, where income stratification leads to a combination of higher tax rates and higher quality of education in richer communities (e.g., Fernandez & Rogerson, 1996; Westhoff, 1977) and agrees with Fernandez and Rogerson (1997). For the moment, ignore the column labeled y_0.

3. Vouchers

3.1. *The inefficiency of an economy without vouchers*

One of the features of the model described in the previous section is that individuals are compelled to consume the education provided by the community in which they live. Such a restriction is likely to lead to an inefficient outcome, an inefficiency that motivates the proposed voucher scheme. A lucid and common criticism of the state of affairs described by the equilibrium in the above model can be found in Friedman (1962), who observed that individuals who live in poor communities get a lower quality of education than those who live in richer communities although they are willing to pay the full cost of education in the latter community. In other words, a poor resident of a poor community may be willing to pay for tuition in better schools but may not be willing to move to the richer community. Children of poor parents may be trapped in low quality schools because it is too costly for their parents to relocate to the other community, and not because they are unwilling to buy better education. To demonstrate such a scenario we shall build a voucher system combined with a price system that leads to a Pareto improvement with respect to the equilibrium defined in the previous section.

Consider an equilibrium $\langle (P, t_P, q_P), (B, t_B, q_B) \rangle$. We want to introduce a voucher program in Providence enabling its residents to choose between sending their children to the local public school or getting the per capita expenditure on education and using it to buy education in Barrington's public school system. Formally, let $e_P = E_P/\lambda(P)$ be the per capita expenditure on public education in Providence. The level of the voucher will be

$$v = e_P. \tag{14}$$

Since we want to show that a Pareto improvement is feasible, we want the quality of education in Barrington not to deteriorate as a result of the flow of newcomers from the neighboring city. To this end, non-Barringtonians who want to use Barrington public schools will be charged a price that allows for the necessary expansion in Barrington's expenditure on education. More formally, letting P_v be the set of Providence inhabitants who choose to use the voucher, the price p charged by Barrington is implicitly defined by the following equation:

$$q_B = \frac{g[E_B + p\lambda(P_v)]}{\lambda(B) + \lambda(P_v)}. \tag{15}$$

This price is chosen to provide the resources necessary to keep the quality of education in Barrington unchanged. Naturally, any difference between the price p and the voucher v must be financed by the individuals who live in Providence and choose to send their children to study in Barrington, out of their own pockets.

The introduction of vouchers brings about some changes in the Providence budget that need to be taken into account. On the one hand, given the tax rate t_P, the total resources available for education are reduced by the amount $v\lambda(P_v)$. On the other hand, less people consume Providence schooling. As a result, the new quality of education in Providence is given by

$$q'_P = \frac{g[E_P - v\lambda(P_v)]}{\lambda(P/P_v)}. \tag{16}$$

Since the expenditure per capita remains the same and g is a concave function with $g(0) = 0$, the quality of education in Providence does not deteriorate: $q'_P \geq q_P$.[5]

Given the equilibrium $\langle (P, t_P, q_P), (B, t_B, q_B) \rangle$ without vouchers, there is a Pareto improvement if we can find a tuition fee p and a subset $P_v \subset P$ of positive measure such that v and p satisfy Eqs. (14)–(16), respectively, and for all $i \in P_v$,

$$u[(1 - t_P)y(i), q'_P] \leq u[(1 - t_P)y(i) + v - p, q_B]. \tag{17}$$

Consider the case of income stratification, with the poor living in Providence and the rich in Barrington:

$$P = y^{-1}([y_l, y_b]) \quad \text{and} \quad B = y^{-1}([y_b, y_h]).$$

Further assume that only the richest inhabitants of Providence use the voucher, namely for some $y_0 \leq y_b$ we have $P_v = y^{-1}([y_0, y_b])$. In this case, Eqs. (15) and (16) become, respectively:

$$q_B = \frac{g\left(E_B + p\int_{y_0}^{y_b} \mathrm{d}F\right)}{\int_{y_0}^{y_h} \mathrm{d}F}, \tag{15$'$}$$

$$q'_P = \frac{g\left(e_P \int_{y_l}^{y_0} \mathrm{d}F\right)}{\int_{y_l}^{y_0} \mathrm{d}F}. \tag{16$'$}$$

The endowment y_0 of an individual who is indifferent between the voucher and Providence public school satisfies

$$u[(1 - t_P)y_0, q'_P] = u[(1 - t_P)y_0 + v - p, q_B]. \tag{17$'$}$$

Consider Fig. 1. It depicts an example of a Pareto improvement. The downward sloping curve, *Price*, represents the function implicitly defined by Eq. (15′), namely, the price p that Barrington residents charge individuals in P_v as a function of y_0, so that their education quality remains unchanged. Fewer people using the vouchers (higher y_0) implies that newcomers will be charged a lower price. The upward sloping curve, *Indif*, represents the price p of the alternative education that makes a person with endowment y_0 indifferent between using the voucher and using the Providence public system. This is implicitly defined by Eq. (17′) after substituting it into Eq. (16′). If these two curves intersect, the intersection point (y_0, p) corresponds to a Pareto

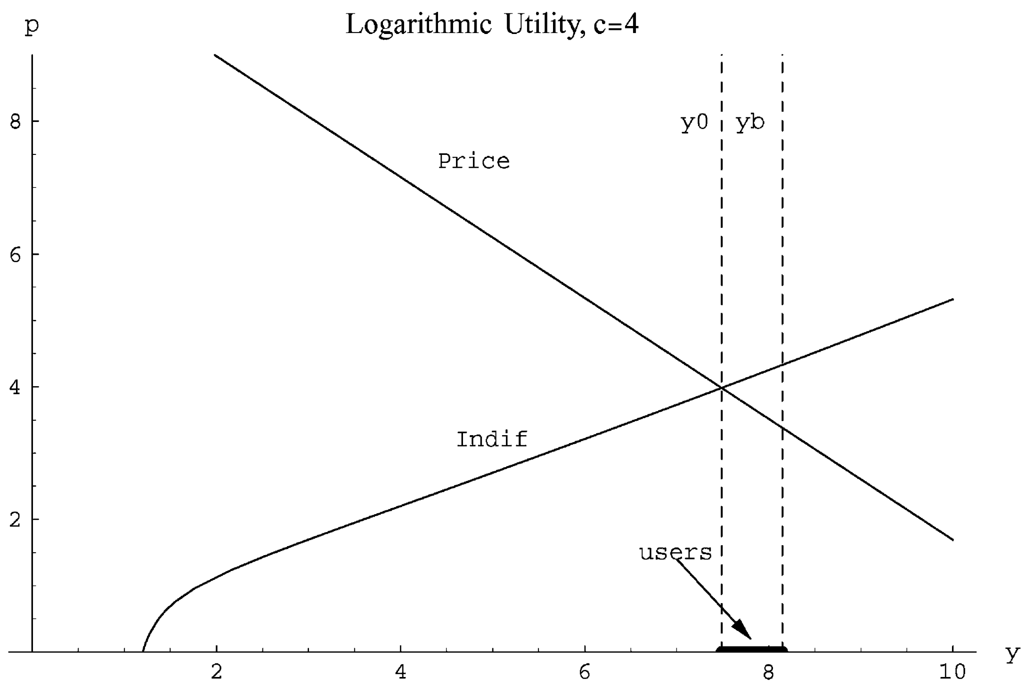

Fig. 1. Vouchers used by the rich in providence.

improvement with respect to the original equilibrium without vouchers: (i) if individuals with endowment in $[y_0, y_b]$ opt for using the vouchers, p is the price necessary to keep the quality of education in Barrington constant; and (ii) if p is the price charged by Barrington residents, then individuals with endowment in $[y_0, y_b]$ are those who are willing to opt for the vouchers.

Reverting to Table 2, it can be seen that when Barrington is exclusive enough, vouchers are used by a certain percentage of the other city's population. The last column, y_0, represents the endowment of a Providence resident who is indifferent between using the voucher and attending a Providence public school. For our example with the logarithmic utility function, the voucher is used when $c > 3$. As this example suggests, the equilibria without vouchers are inefficient.

3.2. Equilibrium with vouchers

In the former section we showed that the equilibrium without vouchers is inefficient. However, our example was not an equilibrium. If a voucher scheme is introduced in Providence, some residents of Barrington might want to move to Providence, so as to save the fixed cost c of living in Barrington while enjoying the same education quality. In this section we look for an equilibrium with vouchers that Pareto dominates the equilibrium without vouchers.

We must first define a community with a voucher system. This will be a group of agents—some of whom will opt to use the voucher while the rest use the local public school—together with a tax rate, a voucher and a quality of education such that (i) both the voucher and the quality of education are financed by proportional income taxation and (ii) the voucher equals the expenditure on education per student.

Definition 3.1. A community with a voucher system (J, J_v, t_J, v, q_J) consists of:

1. A measurable set $J \subset I$ of positive measure of individuals who live in the community.
2. A measurable subset $J_v \subset J$, of individuals who use the voucher, such that $\lambda(J/J_v) > 0$.
3. An income tax rate $t_J \in [0, 1]$.
4. A voucher v such that:

$$v = v(t_J) \equiv \frac{t_J \int_J y(i)\mathrm{d}\lambda}{\lambda(J)}, \tag{18}$$

i.e., the voucher equals per capita expenditure on education.

5. A quality of education q_J such that:

$$q_J = q(t_J, J, J_v) \equiv \frac{g[t_J \int_J y(i)\mathrm{d}\lambda - v\lambda(J_v)]}{\lambda(J/J_v)}, \tag{19}$$

i.e., q_J satisfies the budget constraint.

Next, as in the case of no vouchers, we require that the tax rate be decided by majority vote.

Definition 3.2. Let (J, J_v, t_J, v, q_J) be a community with a voucher system. Let q be the quality of an alternative education system and let p be its price. For each $t \in [0, 1]$, let $J_v(t)$ be the set of agents who use the vouchers and prefer the tax rate t to t_J:

$$J_v(t) = \{i \in J_v : u[(1 - t)y(i) - c_J + v(t) - p, q] > u[(1 - t_J)y(i) - c_J + v - p, q]\}.$$

Similarly, for each $t \in [0, 1]$, let $J_{nv}(t)$ be the set of agents who do not use vouchers and prefer the tax rate t to t_J:

$$J_{nv}(t) = \{i \in J/J_v : u[(1 - t)y(i) - c_J, q(t_J, J, J_v)] > u[(1 - t_J)y(i) - c_J, q_J]\}.$$

We say that (J, J_v, t_J, v, q_J) is a majority voting equilibrium with respect to p and q if

$$\lambda(J_v(t) \cup J_{nv}(t)) \leq \tfrac{1}{2}\lambda(J), \quad \forall t \in [0, 1].$$

We can now define the equilibrium concept that we will use.

Definition 3.3. Let $\mathcal{E} = \langle u, y, g, c\rangle$ be an economy. A voucher equilibrium of $\mathcal{E}$ is a partition of I into a community with a voucher system (P, P_v, t_P, v, q_P) and a viable community (B, t_B, q_B), and a price p that satisfy:

1. Majority voting. (P, P_v, t_P, v, q_P) is a majority voting equilibrium with respect to p and q_B. (B, t_B, q_B) is a majority voting equilibrium in the absorbing community.
2. Non-deterioration of education in the absorbing community:

$$q_B = \frac{g[E_B + p\lambda(P_v)]}{\lambda(B) + \lambda(P_v)}.$$

3. Free mobility. Every agent is happy with his choice of community and education.

(i) $\forall i \in P/P_v$,

$$u[(1-t_P)y(i), q_P] \geq \max\{u[(1 - t_P)y(i) + v - p, q_B], u[(1 - t_B)y(i) - c, q_B]\}.$$

(ii) $\forall i \in P_v$,

$$u\left[(1-t_P)y(i)+v-p,q_B\right] \geq \max\{u\left[(1-t_P)y(i),q_P\right],u\left[(1-t_B)y(i)-c,q_B\right]\}.$$

(iii) $\forall i \in B$,

$$u\left[(1-t_B)y(i)-c,q_B\right] \geq \max\{u\left[(1-t_P)y(i),q_P\right],u\left[(1-t_P)y(i)+v-p,q_B\right]\}.$$

3.3. Example (cont.)

Recall that the common preferences are represented by the utility function $u(x, q) = \ln(x) + \ln(q)$ and that production function of education is $g(z) = z^{\alpha}, 0 < \alpha < 1$. Moreover, the distribution of the initial endowment of the private good is continuous with positive support: $y \in [y_l, y_h] \subseteq \mathbb{R}_{++}$.

We now introduce a proposition that will help us characterize and calculate equilibrium allocations. In an equilibrium where the voucher is no higher than the price charged by the absorbing community, individuals can be partitioned into three consecutive income brackets: the poorest live in Providence and use its public schools, the middle level lives in Providence and uses vouchers and the richest live in Barrington.

Proposition 7. *Let $\langle(P, P_v, t_P, v, q_P), (B, t_B, q_B), p\rangle$ be an equilibrium with vouchers where $p > v$. Then there is income stratification in the following sense:*

1. *$y(i) < y(j)$, for all $i \in P/P_v$ and $j \in P_v$,*
2. *$y(k) < y(l)$, for all $k \in P$ and $l \in B$.*

We denote the borders of the income brackets determined in the previous proposition by y_l, y_v, y_b, and y_h. That is, $[y_l, y_v]$ is the interval of income levels of those Providence residents who do not use the voucher; $[y_v, y_b]$ is the interval of income of those Providence residents who opt for the voucher; and as before, $[y_b, y_h]$ is the income bracket of those individuals who live in Barrington. More graphically, an equilibrium partition should look as shown in Fig. 2.

We now compute equilibria with vouchers and compare them with the corresponding equilibria without vouchers. Since the range of the income function y is convex, it can be shown (as in Proposition 2) that there exists an individual with endowment y_b, who is indifferent between living in Providence or living in Barrington:

$$u[(1-t_P)y_b + v - p, q_B] = u[(1-t_B)y_b - c, q_B]. \tag{20}$$

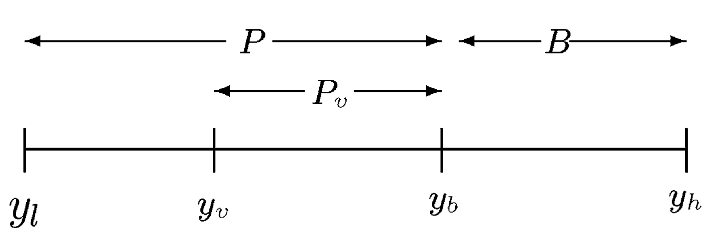

Fig. 2. Partition of agents by income.

By Proposition 7 we know that the richest individuals in Providence use vouchers, and therefore there is a resident of Providence (whose income is y_v) who is indifferent between using the local public school or the voucher:

$$u[(1-t_P)y_v + v - p, q_B] = u[(1-t_P)y_v, q_P]. \tag{21}$$

We now turn to the determination of the communities' tax rates. Consider the community of Providence, (P, P_v, t_P, v, q_P) and an individual $i \in P/P_v$ with initial endowment y. His preferences over taxes are represented by the following function:

$$\ln[(1-t)y] + \ln\left[\frac{g(\lambda(P/P_v)/\lambda(P)\int_P t\, y(i)\mathrm{d}\lambda)}{\lambda(P/P_v)}\right].$$

These preferences are single peaked. The agent's preferred tax rate is:

$$t^*(y) = \frac{\alpha}{1+\alpha}.$$

Consider now an agent $i \in P_v$ with endowment y. Let μ_P be the mean income in community P. Note that the value of the voucher is simply $v_P = t_P\mu_P$. Individual i's preferences over taxes are represented by

$$\ln[(1-t)y + t\mu_P - p] + \ln(q_B),$$

which are single peaked. His preferred tax rate is

$$t^*(y) = \begin{cases} 1 & \text{if } y < \mu_P, \\ 0 & \text{if } y > \mu_P. \end{cases}$$

Since preferences are single peaked, taxes are decided by the median voter. However, the median voter is no longer the agent with median income in the community. From Proposition 7, we know that the relatively rich residents of community P use vouchers. If the agents who use vouchers constitute less than half of the population in the community—an assumption that will be satisfied in our example—then the preferred tax rate of the community's median voter is

$$t_P = \frac{\alpha}{1+\alpha}. \tag{22}$$

Note that assuming that the Providence median voter does not use the voucher, means that the resulting tax rate is independent of the median voter's income. This allows us to calculate the equilibrium without calculating the median voter's income.

Since Barrington does not issue vouchers, the determination of the equilibrium tax rate is as in Section 2.3. Median income in Barrington, y_B^m, is determined by

$$\frac{\int_{y_b}^{y_B^m} \mathrm{d}F}{\int_{y_b}^{y_h} \mathrm{d}F} = \frac{1}{2}, \tag{23}$$

and the corresponding tax rate is

$$t_B = \frac{\alpha}{1+\alpha}\left(1 - \frac{c}{y_B^m}\right). \tag{24}$$

Table 3
Comparing equilibrium with and without vouchers, $c = 4$

	y_b	y_v	t_B	q_B	p	t_P	q_P	v	$q_B - q_P$	q_B/q_P
No voucher	8.1448	–	0.1863	2.8640	–	0.3333	1.3855	–	1.4785	2.0671
Voucher	8.4536	7.6819	0.1888	3.1843	4.3554	0.3333	1.4567	1.5756	1.7276	2.1860

For this table: $u(x, q) = \ln(x) + \ln(q)$, $g(z) = z^\alpha$, $\alpha = 1/2$, $y \sim U[1, 10]$.

By construction, the voucher in Providence is given by its per capita tax revenue:

$$v = \frac{t_P \int_{y_l}^{y_b} y \, \mathrm{d}F}{\int_{y_l}^{y_b} \mathrm{d}F}. \tag{25}$$

On the other hand, the price p is such that new users do not affect the quality of education in Barrington:

$$\frac{g\left(t_B \int_{y_b}^{y_h} y \, \mathrm{d}F + p \int_{y_v}^{y_b} \mathrm{d}F\right)}{\int_{y_v}^{y_h} \mathrm{d}F} = q_B. \tag{26}$$

Given the budget constraint in each community, education quality can be calculated as

$$q_P = \frac{g\left(t_P \int_{y_l}^{y_b} y \, \mathrm{d}F - v \int_{y_v}^{y_b} \mathrm{d}F\right)}{\int_{y_l}^{y_v} \mathrm{d}F}, \qquad q_B = \frac{g\left(t_B \int_{y_b}^{y_h} y \, \mathrm{d}F\right)}{\int_{y_b}^{y_h} \mathrm{d}F}. \tag{27}$$

In order to find an equilibrium, we simply have to solve the former system of nine Eqs. (20)–(27), with nine unknowns: $(y_b, y_v, t_P, y_B^m, t_B, v, p, q_P, q_B)$. Table 3 and Fig. 3 summarize the equilibrium of the numerical example, where $\alpha = 1/2$, $c = 4$ and $y \sim U[1, 10]$.

As anticipated, slightly more people now live in Providence compared with the equilibrium without vouchers (a 3.5% more). A non-trivial proportion of those living in Providence use vouchers: 8.57% of the total population and 10.35% of those who live in Providence. The quality of education increases in both communities. In Providence, the increase in average income and the reduction in the number of users of the public school system dominate the loss

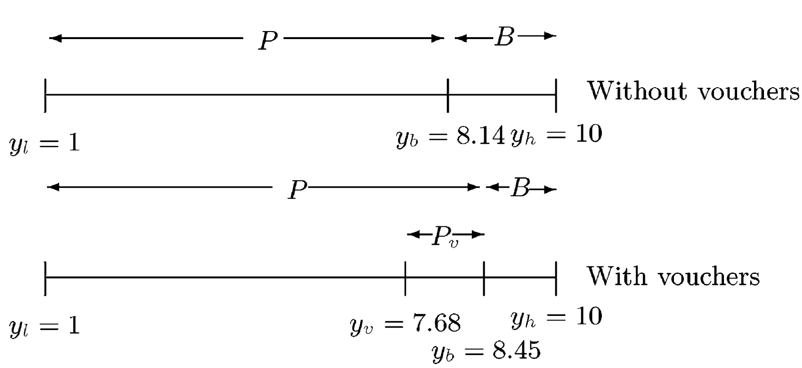

Fig. 3. Equilibrium partitions, $c = 4$.

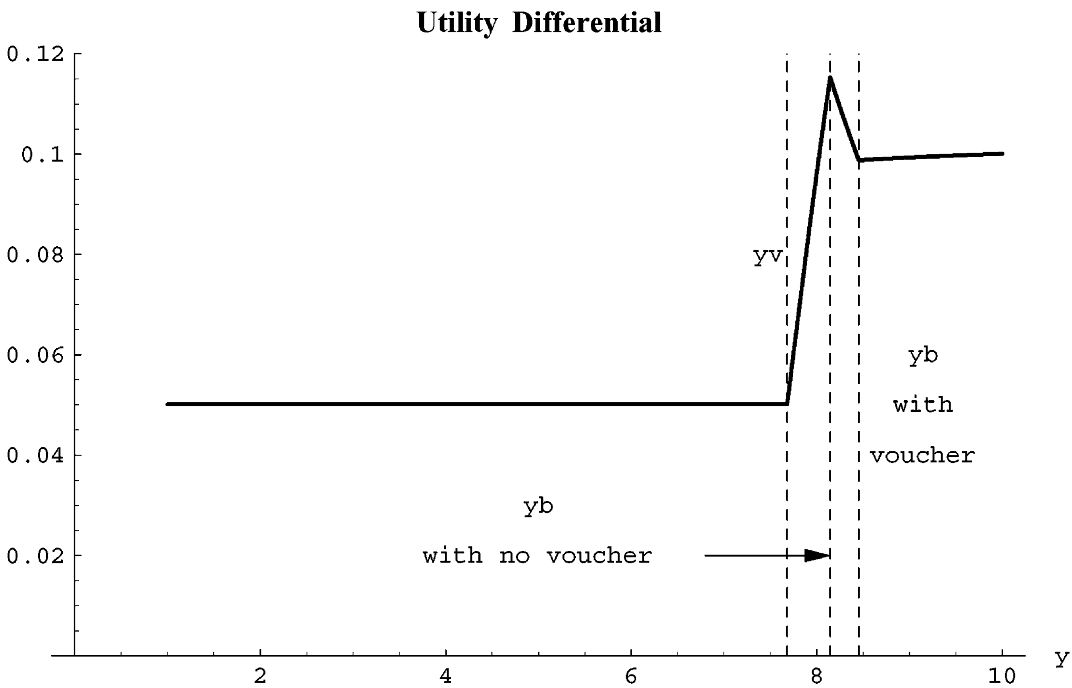

Fig. 4. Equilibrium with vouchers vs. equilibrium without vouchers.

of resources for students using the voucher. In Barrington, education quality increases because the community has become slightly more exclusive and because, by construction, voucher users do not directly affect education quality. Note, however, that the difference in the quality of education provided by the two communities increases in this example.

Fig. 4 depicts the utility differential between the equilibrium with vouchers and the equilibrium without vouchers for all agents in the economy. Utility increases for everybody and therefore the equilibrium with vouchers Pareto dominates the equilibrium without vouchers.

The above qualitative results seem to be robust. When preferences are quasi-linear the same result obtains: vouchers can result in a Pareto improvement as an equilibrium phenomenon. Moreover, in this case there is no income-effect and an equilibrium with the poor of Providence using vouchers is possible.

3.4. *Example with constant returns*

We would like to emphasize that the main results in the former example would obtain in a model with no crowding ($\alpha = 1$) for an appropriate value of the fixed cost, c. The main difference is that if c is either too low or too high in the benchmark model, one of the two communities may be empty. A value of $\alpha < 1$ ensures that no community is empty in equilibrium, but it is not a key determinant of the result that vouchers can result in a Pareto improvement as an equilibrium phenomenon. In order to see this, we present an example with logarithmic utility

Table 4
Comparing equilibrium with and without vouchers, $c = 2.2$

	y_b	y_v	t_B	q_B	p	t_P	q_P	v	$q_B - q_P$	q_B/q_P
No vouchers	7.1046	–	0.3714	3.1762	–	0.5000	2.0262	–	1.15	1.5676
Vouchers	9.1304	7.3652	0.3850	3.6826	3.6826	0.5000	2.5326	2.5326	1.15	1.4540

For this table: $u(x, q) = \ln(x) + \ln(q)$, $g(z) = z^{\alpha}$, $\alpha = 1$, $y \sim U[1, 10]$.

and $\alpha = 1$. Table 4 summarizes equilibrium values for the model with and without vouchers, for $c = 2.2$.

Two things are worth stressing: (1) when $\alpha = 1$, the quality differential between the schools of Providence and Barrington stays constant (i.e., it is the same with and without vouchers); (2) for this particular example, none of the original residents of Providence use vouchers but they are anyway better-off. The graph below shows that the introduction of vouchers is a Pareto improvement (the utility differential is always positive) (Fig. 5).

Rangazas (1995) shows, within a two-community model with private and public education, that vouchers to attend private schools can result in an increase of public education quality in both communities, while the differential of public education across communities decreases. In the examples above, the quality differential increased (decreasing returns) or stayed the same (constant returns). We believe that this fact has to do with the amount of resources left for public education in a community after vouchers are financed rather than with the kind of schools where the vouchers can be used. The larger the proportion of the education expenses that

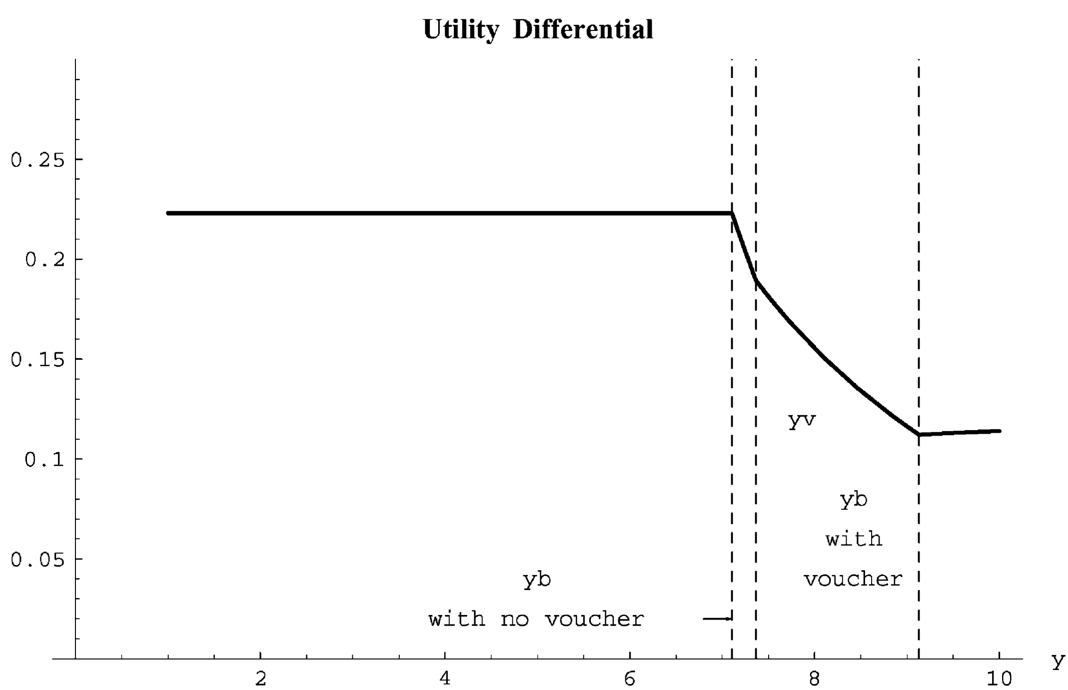

Fig. 5. Equilibrium with voucchers vs. equilibrium without vouchers, $\alpha = 1$.

the voucher finances, the smaller the resources that remain to finance the local public school. Therefore, when the voucher finances a large proportion of the education expenses, as in our model, it is likely that the difference in education quality increases. The smaller the proportion of education expenses that vouchers cover, the more likely that the difference in the quality of education decreases. In Rangazas (1995), the voucher is small compared to average expenditure on public education. In our examples, vouchers are equal to average expenditure on education. Some calculations that we made show that in our model, when the voucher does not cover the full local per capita expenditure in education, the equilibrium difference in education quality decreases.

4. Summary and conclusions

We develop a simple two-good, two-community model that helps us study some of the effects that a voucher system may have on the quality of public education. We show that the introduction of a voucher system may result in a Pareto improvement as an equilibrium outcome. That is, we show not only that the equilibrium of the economy without vouchers is inefficient, but also that the introduction of vouchers in itself, and without the aid of additional money transfers, brings about a state of affairs that is preferred by all members of society.

Needless to say, the model is very simple and abstracts from some important issues but we hope that it provides a clear benchmark for future work. First, we have focused on public education, ignoring private alternatives. Several models consider the consequences of a private sector for education. Without trying to be exhaustive, see for example, Epple and Romano (1996), Glomm and Ravikumar (1998), Ireland (1990) or Stiglitz (1974). Secondly, we have agents who differ only in income, therefore we abstracted from peer group effects. Several authors discuss the importance of peer group effects in education (for example, de-Bartolome, 1990; Epple & Romano, 1998; Henderson, Mieszkowski & Sauvageau, 1978; Manski, 1992). Third, we do not consider housing markets. As explained in the introduction, property taxes are the main source of revenue to pay for local education. However, we choose to use income taxes to keep the model tractable (see Epple & Romer, 1991). Finally, the model that we present is static in nature. Recently Fernandez and Rogerson (1998) have analyzed the dynamic implications of education reforms.

As mentioned above, models abstract from some relevant issues and focus on some others. We hope to have provided a useful benchmark and leave the introduction of private schools, peer group effects and housing markets to future research. This paper has shown that a voucher system that applies only to public schools can result in a Pareto improvement. We believe this result can help in understanding how a voucher system may affect educational outcomes.

Notes

1. Equity issues are not the only arguments cited by voucher opponents. Many think that sending children to religious schools using public money violates the constitutional separation of Church and State. Moreover, some private schools oppose vouchers on

the grounds that such funding will imply more regulation of the private sector. Others are afraid that the equalization of education quality across districts may lead to an equalization of housing prices, losing capitalization rents.

2. Florida has recently approved a voucher system that pays for students to attend higher-rated public schools and not only private schools.
3. Fernandez and Rogerson (1997) introduce zoning restrictions in a model with perfectly elastic housing supply. These restrictions play a similar role to our fixed cost. Rangazas (1995) also considers zoning restrictions.
4. The programs used to produce the results in the paper are available from the authors upon request.
5. Note that $q_P = g(E_P)/E_P v$ and $q'_P = g[E_P - v\lambda(P_v)]/E_P - v\lambda(P_v)v$.

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Appendix A

Proof of Proposition 1. Suppose that there exists an agent i who prefers living in Barrington to living in Providence and let $\tilde{y}$ be his initial endowment. For this agent, $v(\tilde{y}) \leq 0$. If (12) is satisfied, $v(y) < 0, \forall y > \tilde{y}$, which means that all agents richer than i prefer living in Barrington. Similarly, if i prefers Providence to Barrington and if (12) holds, all individuals who are poorer than i also prefer to live in Providence. A similar argument can be used when Eq. (13) holds. □

Proof of Proposition 2. Assume without loss of generality that $y(p) \leq y(b)$ for all $p \in P$ and for all $b \in B$. Define the following income levels: $y_P = \sup\{y(p) : p \in P\}$ and $y_B = \inf\{y(b) : b \in B\}$. Since $B \neq \emptyset$ and $y \geq 0$, y_B is well defined. Since $P \neq \emptyset$ and there is income stratification, y_p is also well defined; moreover, $y_P \leq y_B$. We first prove that $y_P = y_B$. If $y^* \in (y_P, y_B)$, then, since the range of y is convex, there must be an agent i with $y(i) = y^*$. But this agent does not belong to either of the two communities, contradicting the fact that these communities form a partition of the set of agents. Let $y^* = y_P = y_B$ and let i be an agent with $y(i) = y^*$. We claim that i is indifferent between P and B. For $u[(1 - t_P)y^*, q_P] > u[(1 - t_B)y^* - c, q_B]$, we could find a sequence $(y_n)_{n\in\mathbb{N}}$ of income levels in $\{y(b) : b \in B\}$ that converges to $y_B = y^*$ and such that $u[(1 - t_P)y_n, q_P] \leq u[(1-t_B)y_n - c, q_B]$, for all n. Then, by continuity of u we would have $u[(1-t_P)y^*, q_P] \leq u[(1-t_B)y^* - c, q_B]$, which is a contradiction. An analogous argument shows that i cannot prefer B over P. □

Proof of Proposition 3. Let $\mu(J) = \int_J y(i)\mathrm{d}\lambda$. An agent who lives in community J has the following indirect utility function:

$$V = u\left[(1-t)y - c_J, \frac{g(t\mu(J))}{\lambda(J)}\right]. \tag{A.1}$$

Since u and g are continuous functions and [0, 1] is a compact set, V has a maximum there. It is enough, then, to show that V is a strictly concave function of t, as in this case there is a unique maximizer which is the single peak. It can be shown that

$$\frac{\mathrm{d}^2 V}{\mathrm{d}t^2} = y^2 u_{11} - 2y\frac{\mu(J)}{\lambda(J)}g' u_{12} + \left(\frac{\mu(J)}{\lambda(J)}\right)^2 (g')^2 u_{22} + \frac{\mu(J)^2}{\lambda(J)} u_2 g'' \tag{A.2}$$

which, given our assumptions, is negative and therefore V is strictly concave. □

Proof of Proposition 4. Note that u_2/yu_1 is simply the slope of the indifference curve of an agent with initial endowment y in the (q, t) space. If (q_J, t_J) is the most preferred quality-tax pair for agent i, then the assumption that the slope is monotonic in income implies that for each tax rate t, either the individuals who are richer than him or the individuals who are poorer than him prefer t_J to t. When i is the individual with median income, this implies that the community is in a majority voting equilibrium. □

Proof of Proposition 5.

1. Follows from Proposition 3.
2. Since preferences are single peaked, by the median voter theorem the majority voting equilibrium in each community is given by the median voter. It can be shown that the slope of the indifference curves in the (q, t) space is
$$\frac{u_2}{yu_1} = \frac{1-t}{q} - \frac{c}{qy}. \tag{A.3}$$
Since this slope is a monotonic function of y, by Proposition 4, the median voter is the agent with median income.
3. Let $v(y)$ denote the utility differential between living in Providence and living in Barrington for an agent with endowment y,
$$v(y) = \ln[(1-t_P)y] + \ln(q_P) - \ln[(1-t_B)y - c] - \ln(q_B). \tag{A.4}$$
By Proposition 1, it is enough to show that $\mathrm{d}v(y)/\mathrm{d}y < 0$. And indeed,
$$\frac{\mathrm{d}v(y)}{\mathrm{d}y(i)} < 0 \Leftrightarrow \frac{1-t_P}{(1-t_P)y} - \frac{1-t_B}{(1-t_B)y - c} < 0 \Leftrightarrow -(1-t_P)c < 0, \tag{A.5}$$
which is true since $c > 0$ and given the utility function, in equilibrium we must have $t_P < 1$.
4. Follows from Proposition 2 and the fact that the range of y is convex. □

Proof of Proposition 6.

1. Let $\langle(P, t_P, q_P), (B, t_B, q_B)\rangle$ be an equilibrium. Consider the community of Barrington (B, t_B, q_B). By Proposition 5, t_B must be the preferred tax rate of the agent with median

income in Barrington. Letting y_B^m be the median voter's income, his preferences over tax rates are represented by

$$\ln[(1-t)y_B^m - c] + \ln\left[\frac{\left(t\int_B y(i)\,\mathrm{d}\lambda\right)^{\alpha}}{\lambda(B)}\right]. \tag{A.6}$$

Consequently, his preferred tax rate is given by

$$t_B = \frac{\alpha}{1+\alpha}\left(1 - \frac{c}{y_B^m}\right). \tag{A.7}$$

Similarly, since (P, t_P, q_P) is a majority voting equilibrium of Providence, t_P must be the preferred tax rate of the agent with median income in Providence.

This optimal tax is given by

$$t_P = \frac{\alpha}{1+\alpha}. \tag{A.8}$$

Since $c > 0$ and $y_B^m > 0$, we conclude that $t_B < t_P$.

2. By Proposition 5, there is an individual who is indifferent between living in Providence and living in Barrington. Denote his endowment by $y_b = y(i_b)$.

$$\ln[(1-t_P)y_b] + \ln(q_P) = \ln[(1-t_B)y_b - c] + \ln(q_B) \Leftrightarrow \frac{q_B}{q_P} = \frac{(1-t_P)y_b}{(1-t_B)y_b - c}.$$

Substituting for t_P and t_B from Eqs. (A.7) and (A.8), we obtain:

$$\frac{q_B}{q_P} = \frac{y_b}{y_b + \alpha c(y_b/y_B^m) - \alpha c - c}, \tag{A.9}$$

which is bigger than 1 since, by Proposition 5 $(y_b/y_B^m) < 1$. Therefore, $q_B > q_P$. □

Proof of Proposition 7.

1. Let

$$w(y) = \ln[(1-t_P)y] + \ln(q_P) - \ln[(1-t_P)y + v - p] - \ln(q_B), \tag{A.10}$$

i.e., the utility differential between living in community P and using its public schools, and living in community P and using the voucher to go to school in community B. The result follows after noting that

$$\frac{\mathrm{d}w(y)}{\mathrm{d}y} = \frac{v-p}{[(1-t_P)y - v + p]y} < 0$$

for all $y \in [y_l, y_h]$.

2. From the previous item we know that the richest individuals in Providence use the voucher. Let

$$v(y) = \ln[(1-t_P)y + v - p] + \ln(q_B) - \ln[(1-t_B)y - c] - \ln(q_B)$$

be the utility differential between living in Providence using the voucher and living in Barrington. The result follows after noting that

$$\frac{\mathrm{d}v(y)}{\mathrm{d}y} = \frac{1-t_P}{(1-t_P)y+v-p} - \frac{1-t_B}{(1-t_B)y-c}$$
$$= \frac{-c(1-t_P)-(v-p)(1-t_B)}{[(1-t_B)y-c][(1-t_P)y+v-p]} < 0. \qquad \square$$

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